Solution to MHT CET – 2021
22nd September (Shift - 1)

Section I

PHYSICS

1. (D)

Kinetic energy $= \frac{L_P^2}{2I_P} = \frac{L_Q^2}{2I_Q}$

$\therefore \quad \frac{L_Q^2}{L_P^2} = \frac{I_Q}{I_P}$

$\because \quad I_Q > I_P$ we get $L_Q > L_P$

2. (A)

Phase difference $\phi = \frac{2\pi}{\lambda}x$

$$\lambda = \frac{v}{f} = \frac{330}{50} = 6.6$$

$$\therefore \quad x = \frac{\lambda\phi}{2\pi} = \frac{6.6}{2\pi} \times \frac{\pi}{3} = \frac{6.6}{6} = 1.1\,\text{m}$$

3. (D)

X–OR gate gives 'High' output only when one input is high and the other is low.

4. (C)

Magnetic field at the centre of the coil is given by

$$B = \frac{\mu_0 NI}{2R}$$

Magnetic flux linked to the coil is

$$\phi = NBA = \frac{\mu_0 N^2 I}{2R} \cdot \pi R^2 = \frac{\mu_0 N^2 \pi RI}{2}. \quad \text{[Assuming field is uniform]}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 \pi R}{2}.$$

5. (D)

Initial current $I = \frac{V}{z}$, Final current $I' = \frac{V}{z}$

$\frac{1}{2} = \frac{I'}{I} = \frac{z}{z'} \qquad \therefore \quad \frac{z}{z'} = \frac{1}{2}$

$\therefore \quad \frac{z^2}{z'^2} = \frac{1}{4} \qquad \therefore \quad \frac{R^2 + X_c^2}{R^2 + X_c'^2} = \frac{1}{4}$

$\therefore \quad 4R^2 + 4X_c^2 = R^2 + X_c'^2$

$\therefore \quad 3R^2 = X_c'^2 - 4X_c^2 \qquad \ldots(1)$

$X_c = \frac{1}{\omega C}, \quad X' = \frac{3}{\omega C} \quad \therefore X' = 3X_c$

Putting this value of X′ in (1) we get

$$3R^2 = 9X_c^2 - 4X_c^2 = 5X_c^2$$

$$\therefore \frac{X_c^2}{R^2} = \frac{3}{5} = 0.6 \quad \therefore \quad \frac{X_c}{R} = \sqrt{0.6}$$

6. (D)

Comparing with the standard equation

$$y = A\sin(kx + \omega t)$$

we get $k = 0.01$ rad/cm and $\omega = 30$ rad/s

Speed of the wave $v = \frac{\omega}{k} = \frac{30}{0.01} = 3000$ cm/s = 30 m

$\therefore$ Distance = vt = 30 × 0.5 = 15 m

7. (B)

Kinetic energy is directly proportional to absolute temperature.

8. (C)

$$(K.E.)_1 = \frac{hc}{\lambda_1} - W_0$$

Multiplying by 3/2

$$\frac{3}{2}(K.E.)_1 = \frac{3}{2}\frac{hc}{\lambda_1} - \frac{3}{2}W_0 \quad \ldots(1)$$

$$(K.E.)_2 = \frac{hc}{\lambda_2} - W_0$$

$$(K.E.)_2 = \frac{3}{2}\frac{hc}{\lambda_1} - W_0 \quad \ldots(2) \quad \left(\because \lambda_2 = \frac{2}{3}\lambda_1\right)$$

By equation (1) and (2)

$$(K.E.)_2 = \frac{3}{2}(K.E.)_1 \quad \text{or} \quad \frac{1}{2}mv_2^2 > \frac{3}{2}\left(\frac{1}{2}mv_1^2\right)$$

$$\therefore v_2^2 = \frac{3}{2}v_1^2$$

$$\therefore v_2 = (1.5)^{\frac{1}{2}} v_1$$

9. (D)

For adiabatic expression, we have

$$P_2V_2^{\gamma} = P_1V_1^{\gamma}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (8)^{\frac{4}{3}} = 16$$

$$\therefore P_2 = 16\,P_1 = 16\,P_0$$

10. (D)

If ω_1 is angular velocity of hour hand of clock and ω_2 is the angular velocity of the earth, then the ratio of $\omega_1 : \omega_2$ is

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = \frac{24\text{ hours}}{12\text{ hours}} = 2$$

11. (B)

Emf induced $e = B\ell v = 0.5 \times 1 \times 2 = 1$ V

Rate of doing work = Power $P = \frac{e^2}{R} = \frac{(1)^2}{6} = \frac{1}{6}$ W

12. (A)

$Y_1 = \frac{10\lambda_1 D}{d}$, $Y_2 = \frac{5\lambda_2 D}{d}$

$\therefore \frac{Y_1}{Y_2} = \frac{10\lambda_1}{5\lambda_2} = \frac{2\lambda_1}{\lambda_2}$

13. (B)

$m = 0.4$ kg, $f = \frac{16}{\pi}$ Hz, K.E. = 2 J, P.E. = 1.2 J

$\omega = 2\pi f = 2\pi \times \frac{16}{\pi} = 32$ rad/s

Total energy T.E. = 2 + 1.2 = 3.2 J

$TE = \frac{1}{2} m\omega^2 A^2$

$\therefore A^2 = \frac{2(TE)}{m\omega^2} = \frac{2 \times 3.2}{0.4 \times (32)^2} = \frac{6.4}{0.4 \times (32)^2}$

$= \frac{16}{(32)^2}$

$\therefore A = \frac{4}{32} = \frac{1}{8} = 0.125$ m

14. (A)

$V_e = \sqrt{\frac{2GM}{R}}$; $V_p = \sqrt{\frac{2GM_p}{R_p}}$

$\therefore \frac{V_p}{V_e} = \sqrt{\frac{M_p}{M} \cdot \frac{R}{R_p}} = \sqrt{6 \times \frac{1}{2}} = \sqrt{3}$

$\therefore V_p = \sqrt{3}\, V_e$

15. (C)

For a biconvex lens

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= (1.6 - 1)\left(\frac{1}{30} + \frac{1}{30}\right)$

$= 0.6 \times \frac{2}{30} = \frac{1.2}{30}$

$f = \frac{30}{1.2} = 2.5$ cm

Radius of concave mirror = 2f = 50 cm

16. (B) **17.** (B) **18.** (B)

19. (D)
For a vibrating wire, we have Tp^2 = constant
where p is the number of loops formed and T is the tension

$T_1 = 9$ kg–wt, $p_1 = 5,\ p_2 = 3$

$\therefore\ T_1P_1^2 = T_2P_2^2$

$\therefore\ 9 \times 25 = T_2 \times 9$

$\therefore\ T_2 = 25$ kg–wt

20. (D)
Rate of energy emission $R \propto T^4$

$$\therefore\ \frac{R_2}{R_1} = (2)^4 = 16$$

21. (B)

$$h \propto \frac{1}{r}\quad \therefore\ \frac{h_2}{h_1} = \frac{r_1}{r_2} = 3$$

$$\therefore\ h_2 = 3h_1$$

$$m_1 = h_1\pi r_1^2\rho,\ \ m_2 = h_2\pi r_2^2\rho$$

$$\therefore\ \frac{m_2}{m_1} = \frac{h_2}{h_1}\left(\frac{r_2}{r_1}\right)^2 = 3\times\left(\frac{1}{3}\right)^2 = \frac{1}{3}$$

$$\therefore\ m_2 = \frac{m_1}{3}$$

22. (C)
For balanced Wheatstone's bridge we have

$$\frac{P}{Q} = \frac{R}{S}$$

where S is the equivalent resistance of S_1 and S_2 in parallel

$$S = \frac{S_1S_2}{S_1+S_2}$$

$$\therefore\ \frac{P}{Q} = \frac{R(S_1+S_2)}{S_1S_2}$$

23. (A)

$$E_1 = \frac{1}{2}LI_1^2,\ E_2 = \frac{1}{2}LI_2^2$$

$$\therefore\ \frac{E_2}{E_1} = \left(\frac{I_2}{I_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\therefore\ E_2 = \frac{E_1}{4}$$

24. (D)

$$_{B}\mu_{A} = \frac{v_B}{v_A} = \frac{1}{\sin\theta}$$

25. (B)

We have $\Delta Q = \Delta U + \Delta W$

In an adiabatic process, $\Delta Q = 0$. $\quad \therefore \Delta U = -\Delta W$

26. (A)

The frequency of LC oscillation is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \frac{f_2}{f_1} = \sqrt{\frac{c_1}{c_2}} = \sqrt{\frac{0.1}{0.2}} = \frac{1}{\sqrt{2}}$$

$$\therefore f_2 = \frac{f_1}{\sqrt{2}}$$

27. (B)

Moment of inertial of the sphere $I_1 = \frac{2}{5}MR^2$

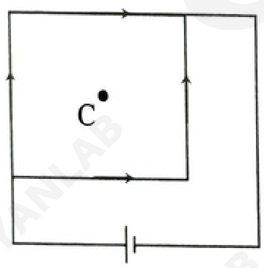
Moment of inertia of the cylinder $I_2 = \frac{1}{2}MR^2$

$$\therefore \frac{I_1}{I_2} = \frac{4}{5}, \frac{\omega_1}{\omega_2} = \frac{1}{2}$$

K.E. $k = \frac{1}{2}I\omega^2$

$$\therefore \frac{k_1}{k_2} = \frac{I_1}{I_2}\left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{4}{5} \times \left(\frac{1}{2}\right)^2 = \frac{1}{5}$$

28. (D)



As shown in the figure, the currents in the parallel sides will produce equal and opposite fields at the centre and hence the net magnetic field at the centre will be zero.

29. (A)

Initial kinetic energy $k_1 = \frac{1}{2}mu_1^2$

Final kinetic energy $k_2 = \frac{1}{2}mu_2^2 = \frac{1}{2}m\left(2u_1^2\right) = \frac{1}{2}\left(4mu^2\right)$

$$\therefore k_2 - k_1 = \frac{3}{2}mu^2$$

Change in K.E. is work done

Power P $\frac{\text{work done}}{t} = \frac{3mu^2}{2t}$

30. (D)
When it is displaced through 90° from mean position, it is at a height 'r' and has potential energy mgr. At the lowest position this potential energy is converted into kinetic energy.

$\therefore \frac{1}{2}mv^2 = mgr$

$\therefore mv^2 = mgr$

At the lowest position the tension in the string

$$T = mg + \frac{mv^2}{r} = mg + 2mg = 3mg$$

31. (D)
$W_1 = 8\pi r^2 T$
where T = surface tension
$W_2 = 8\pi(2r)^2 T' = 32\, r^2 T'$
Since soap solution is heated, its surface tension decreases.
$\therefore T' < T$
$\therefore W_2 < 4W_1$ or $4W_1 > W_2$

32. (C)
$$Q = \frac{kAd\theta}{d}$$
If the radius of the cylindrical rod is doubled, then its area of cross–section will become four times.
$\therefore A_2 = 4A_1$
Also $d_2 = d_1$
$$\therefore \frac{Q'}{Q} = \frac{A_2}{A_1} \cdot \frac{d_1}{d_2} = 4 \times \frac{1}{2} = 2$$
$\therefore Q' = 2Q$

33. (B)
Magnetic field due to current i_1 is given by
$$B_1 = \frac{\mu_0 i_1}{2r_1}$$
Similarly, $B_2 = \frac{\mu_0 i_2}{2r_2}$

The resultant field at the centre $B = B_1 - B_2$

It is given that $B = \frac{B_1}{2}$

$$\therefore \frac{B_1}{2} = B_1 - B_2$$
$$\therefore B_2 = \frac{B_1}{2}$$
$$\therefore \frac{\mu_0 i_2}{2r_2} = \frac{1}{2}\left(\frac{\mu_0 i_1}{2r_1}\right) \qquad \therefore \frac{i_2}{r_2} = \frac{1}{2} \cdot \frac{i_1}{r_1}$$
$$\therefore \frac{i_2}{i_1} = \frac{1}{2} \cdot \frac{r_2}{r_1} = \frac{1}{2} \cdot 2 = 1$$

34. (A)

35. (A)

$\mu = 0.1256,$

$\mu_0 = 4\pi \times 10^{-7} = 4 \times 3.14 \times 10^{-7}$

$= 12.56 \times 10^{-7}\ \text{TmA}^{-1}$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.1256}{12.56 \times 10^{-7}} = 10^5$$

36. (D)

$$\theta = 5\sin\frac{\pi t}{6}$$

$$\therefore \frac{d\theta}{dt} = \frac{\pi}{6}\cdot 5\cos\frac{\pi t}{6}$$

$$\text{At } t = 3\text{ s, } \frac{d\theta}{dt} = 5\cos\frac{\pi \times 3}{6} = \frac{5\pi}{6}\cos\frac{\pi}{2} = 0 \qquad \left(\cos\frac{\pi}{2} = 0\right)$$

37. (A)

Distance of nth maximum $\quad y_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{a}$

$$\therefore y_2 = \left(2 + \frac{1}{2}\right)\frac{\lambda_1 D}{a} \text{ and } y_3 = \left(3 + \frac{1}{2}\right)\frac{\lambda_2 D}{a}$$

$$\therefore 2.5\lambda_1 = 3.5\lambda_2$$

$$\therefore 2.5 \times 6000 = 3.5\lambda_2$$

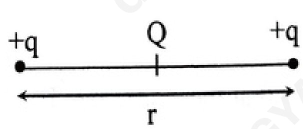
$$\therefore \lambda_2 = \frac{2.5 \times 6000}{3.5} \simeq 4300\ \text{Å}$$

38. (B)

The forces on charge Q due to the other two charges will be equal and opposite and hence it will be in equilibrium.

Force on +q due to the other two charges should also be equal and opposite. Their magnitudes will be equal if

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{\left(\frac{r}{2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qQ}{r^2}$$



$$\therefore q = 4Q \quad \text{or} \quad Q = \frac{q}{4}$$

Q and q should have opposite signs so that the two forces are opposite in direction.

$$\therefore Q = -\frac{q}{4}$$

39. (C)

Energy stored is given by $\quad E = \frac{q^2}{2C}$

$$\therefore \frac{dE}{dq} = \frac{2q}{2C} = \frac{q}{C} \qquad \therefore dE = \frac{q}{C}\cdot dq$$

$\therefore \frac{dE}{E} = \frac{2dq}{q}$

$\therefore 0.21 = \frac{2dq}{q} = \frac{2\times 2}{q} = \frac{4}{q} \quad [\because dq = 2C]$

$\therefore q = \frac{4}{0.21} \simeq 20\,C$

40. (C)
By law of conservation of momentum, the two parts will have equal and opposite momentum. assuming the nucleus was initially at rest.

$\therefore m_1 v_1 = m_2 v_2$ or $\frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{1}{2}$

$\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{2}$

$\therefore \frac{r_1}{r_2} = \frac{1}{2^3}$

41. (B)
Centripetal acceleration $a = r\omega^2 = r(2\pi n)^2$
$= 4\pi^2 n^2 r$

42. (A)
For 6 degrees if freedom, $C_v = 6 \times \frac{1}{2} RT = 3RT$

$C_p = C_v + R = 4RT$

$\therefore \frac{C_p}{C_v} = \frac{4}{3}$ or $C_p = \frac{4}{3} C_v$

$R = C_p - C_v = \frac{4}{3} C_v - C_v = \frac{C_v}{3}$

43. (D)
Since the battery is disconnected, the charge on the capacitor remains constant. When the dielectric slab is introduced, the capacitance increases k times.

Energy stored $= \frac{q^2}{2C}$ since C increases, energy decreases.

44. (A)
The width of central maximum is given by

$W = \frac{2\lambda D}{a}$

W depends on wavelength of light λ, which is related to the frequency. Hence question is not proper.

45. (D)
The period of the S.H.M. does not depend on the amplitude. Its energy is given by

$E = \frac{1}{2} m\omega^2 A^2$

Hence its energy will decrease if amplitude is decreased.

46. (B)

$q = 200 \times 1.6 \times 10^{-19}$ C, $t = 10^{-8}$ s

$\therefore$ Emitter current $I_e = \frac{q}{t} = \frac{3.2\times10^{-17}}{10^{-8}} = 3.2\times10^{-9}$ A

$I_b = \frac{1}{100}\cdot I_e$

$I_c = \frac{99}{100}\cdot I_e$

$\therefore$ Current amplification factor $\beta = \frac{I_c}{I_b} = 99$

47. (C)

Capacitors C_2 and C_3 are in parallel. Hence their equivalent capacitance

$C_4 = C_2 + C_3 = 8 + 4 = 12\,\mu F$

C_4 and C_1 are in series.

Their equivalent capacitance $C = \frac{12\times6}{12+6} = \frac{72}{18} = 4\,\mu F$

Charge stored by the combination $q = CV = 4 \times 900$
$= 3600\,\mu C$

In the series combination, change in same on each capacitor.

$\therefore$ Charge on $C_1 = 3600\,\mu C$

P,D, across C_1 is $V_1 = \frac{q}{C_1} = \frac{3600}{6} = 600$ V

$\therefore$ $V_A - V_P = 600$ $\therefore$ $900 - V_P = 600$

$\therefore$ $V_P = 300$ V

48. (A)

If E is the kinetic energy of the proton, then

$E = \frac{p^2}{2m}$ where p is the momentum and m is the mass of proton

$\therefore$ $p = \sqrt{2mE}$ $\therefore$ $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

For photon, $E = \frac{hc}{\lambda_2}$ $\therefore$ $\lambda_2 = \frac{hc}{E}$

$\therefore$ $\frac{\lambda_1}{\lambda_2} = \frac{h}{\sqrt{2mE}}\cdot\frac{E}{hc} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

$\therefore$ $\frac{\lambda_1}{\lambda_2} \propto E^{1/2}$ $\therefore$ $n = \frac{1}{2}$

49. (D)

The drop is in equillibrium under the action of the following forces :

Weight of the liquid = $Mg = \frac{4}{3}\pi r^3 \rho g$ (downwards)

Upthrust = weight of the liquid displaced

$\therefore\ U = \frac{4}{6}\pi r^3 dg$ (upwards)

(Since the drop is half immersed, the volume of the liquid displaced is half the volume of the drop)

Force due to surface tension $F = 2\pi rT$ (upwards)

$\therefore\ Mg = F + U$

$\therefore\ F = Mg - U$

$$\therefore\ 2\pi rT = \frac{4}{3}\pi r^3\rho g - \frac{4}{6}\pi r^3 dg$$

$$\therefore\ T = \frac{2}{3}r^2\rho g - \frac{1}{3}r^2 dg$$

$$= r^2 g\left(\frac{2}{3}\rho - \frac{1}{3}d\right)$$

$$= r^2 g\left(\frac{2\rho - d}{3}\right)$$

$$\therefore\ r^2 = \frac{3T}{g(2\rho - d)}$$

If D is the diameter of the drop then $r^2 = \frac{D^2}{4}$

$$\therefore\ \frac{D^2}{4} = \frac{3T}{g(2\rho - d)}$$

$$\therefore\ D^2 = \frac{12T}{g(2\rho - d)}$$

$$\therefore\ D = \sqrt{\frac{12T}{g(2\rho - d)}}$$

50. (A)

The magnetic field at the centre of a circular coil is given by

$$B = \frac{\mu_0 I}{2r}$$

If T is the periodic time of revolving electron, then $I = \frac{e}{T}$

Also, $T = \frac{2\pi r}{V}$

$$\therefore\ I = \frac{eV}{2\pi r}$$

$$\therefore\ B = \frac{\mu_0 eV}{4\pi r^2} \quad \ldots(1)$$

For electron in ground state,

$$V = \frac{e^2}{2\epsilon_0 h} \text{ and } r = \frac{h^2 \epsilon_0}{\pi m e^2}$$

Putting these values in eq. (1) and simplifying we get

$$\therefore\ B = \frac{\mu_0 e^7 \pi m^2}{8\epsilon_0 h^5}$$

CHEMISTRY

51. (D)

1 mole of substance $= 6.022 \times 10^{23}$ atoms

$\therefore$ 0.25 mole of substance $= 0.25 \times 6.022 \times 10^{23}$ atoms

$= 1.5055 \times 10^{23}$ atoms

The number of octahedral voids, $N = 1.5055 \times 10^{23}$

The number of tetrahedral voids, $2N = 3.011 \times 10^{23}$

52. (D)

$k = 2 \times 10^{-2}\ s^{-1}$, $[A]_0 = 1.0$ mol dm^{-3}, $t = 100$ s, $\log \frac{1}{[A]_t} = ?$

For first order reaction,

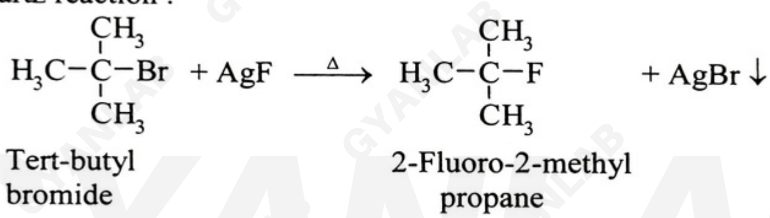
$$k = \frac{2.303}{t} \log_{10} \frac{[A]_0}{[A]_t}$$

$$\therefore \log_{10} \frac{[A]_0}{[A]_t} = \frac{k \times t}{2.303}$$

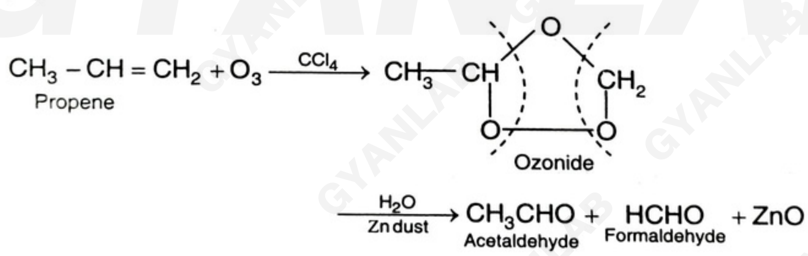
$$\therefore \log_{10} \frac{1}{[A]_t} = \frac{2 \times 10^{-2} \times 100}{2.303} = 0.868 \text{ mol dm}^{-3}$$

53. (B)

Swartz reaction :



54. (D)



55. (B)

$\Delta P = 2.5$ mm Hg, $P_1^0 = 250$ mm Hg

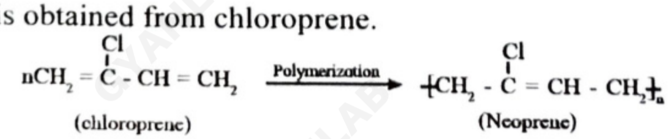
$$x_2 = \frac{\Delta P}{P_1^0} = \frac{2.5}{250} = 0.01$$

56. (B)

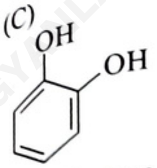
Oxygen family (Gr. 16 elements) : O, S, Se, Te, Po

57. (B)

It is obtained from chloroprene.



58. (C)



IUPAC name : Benene-1,2-diol
Common name : Catechol

59. (B)

60. (A)
Mn (Z = 25) $\rightarrow$ [Ar] $3d^5\ 4s^2$
Manganese shows oxidation states from +2 to +7.

61. (B)
The order of boiling points of alkanes, amines, alcohols and carboxylic acids of comparable molar mass is as follows :
Alkanes < Amines < Alcohols < Carboxylic acids

62. (B)
$K_a = 2.7 \times 10^{-5},\ \propto = 3 \times 10^{-2},\ c = ?$

$K_a = \propto^2 c$

$\therefore\ c = \dfrac{K_a}{\propto^2} = \dfrac{2.7\times10^{-5}}{(3\times10^{-2})^2} = \dfrac{2.7\times10^{-5}}{9\times10^{-4}} = 0.03\ M$

63. (D)
For first order reaction, the unit of rate constant is $(\text{time})^{-1}$.

64. (D)

65. (A)
Molar mass of methane (CH_4) = 16 g mol^{-1}
For 16 g of CH_4, enthalpy of formation is = –75 kJ mol^{-1}

$\therefore$ For 24 g of CH_4, enthalpy change for formation $= \dfrac{24\times(-75)}{16} = -112.5$ kJ

66. (C)
$K_{sp} = 3.2 \times 10^{-8},\ \ S = ?$

$AX_2 \rightleftharpoons A^{2+} + 2X^{2-}$

$\therefore\ K_{sp} = 4S^3$

$\therefore\ S^3 = \dfrac{K_{sp}}{4} = \dfrac{3.2\times10^{-8}}{4} = 0.8 \times 10^{-8}$

$\therefore\ S = \sqrt[3]{8\times10^{-9}} = 2 \times 10^{-3}$

67. (D)

68. (C)

$$2\ H_3C-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[\text{Dimethyl cadmium}]{(CH_3)_2Cd} 2\ H_3C-\overset{O}{\overset{\|}{C}}-CH_3 + CdCl_2$$

Ethanoyl Chloride — Propanone Chloride — Cadmium

69. (B)

$k = 200\ ohm^{-1}\ cm^{-1}$, cell constant $= 1\ cm^{-1}$

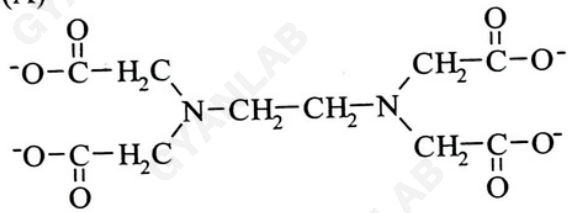
$$R_{Solution} = \frac{\text{cell constant}}{k_{KCl}} = \frac{1\ cm^{-1}}{200\ ohm^{-1}\ cm^{-1}} = 5 \times 10^{-3}\ ohm$$

70. (D)

71. (A)

Glutaric acid	Malonic acid
$COOH-(CH_2)_3-COOH$	$COOH-CH_2-COOH$

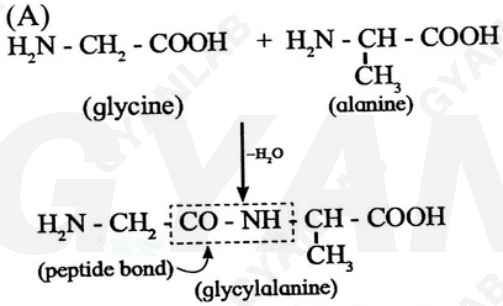
72. (A)



Ethylenediaminetetraacetate ion

Two N atoms are present in EDTA.

73. (A)



In the dipeptide glycylalanine, glycine residue is N-terminal and alanine residue is C-terminal.

74. (A)

75. (B)

$c = 0.04\ M,\ \wedge = 230\ \Omega^{-1}\ cm^2\ mol^{-1},\ k = ?$

$$\wedge = \frac{1000\,k}{c}$$

$$\therefore\ k = \frac{\wedge c}{1000} = \frac{230\,\Omega^{-1} cm^2 mol^{-1} \times 0.04\,mol\,L^{-1}}{1000\,cm^3\,L^{-1}} = 9.2 \times 10^{-3}\ \Omega^{-1}\ cm^{-1}$$

76. (D)

$P_1 = 210\ kPa,\ P_2 = 105\ kPa,\ T = 300\ K,$

$n = 1,\ R = 8.314\ J\ K^{-1}\ mol^{-1},\ W = ?$

$$W_{max} = -2.303\ nRT\ \log_{10}\frac{P_1}{P_2} = -2.303 \times 1 \times 8.314 \times 300 \times \log_{10}\frac{210}{105}$$

$$= -2.303 \times 1 \times 8.314 \times 300 \times \log_{10} 2 = -2.303 \times 1 \times 8.314 \times 300 \times 0.3010$$

$$= 1728.99\ J$$

$$\therefore\ W_{max} = 1729\ J$$

77. (C)

$$KClO_3 \longrightarrow KCl + \frac{3}{2} O_2$$

122.5 g   74.5 g

122.5 g of $KClO_3$ produces 74.5 g of KCl

$\therefore$ 12.25 g of $KClO_3$ produces $= \dfrac{74.5 \times 12.25}{122.25} = 7.45$ g of KCl

78. (B)

79. (B)

$\rho = 8.57 \text{ g cm}^{-3}$, $a = 3.3 \text{ Å} = 3.3 \times 10^{-8}$ cm

Packing efficiency = 68%

$\therefore$ It is bcc type of lattice. Hence, n = 2

$$M = \frac{\rho \times a^3 \times N_A}{n}$$

$$= \frac{8.57 \text{ g cm}^{-3} \times (3.3 \times 10^{-8})^3 \text{ cm}^3 \times 6.022 \times 10^{23} \text{ atoms mol}^{-1}}{2 \text{ atoms}}$$

$$= \frac{8.57 \times 35.937 \times 10^{-24} \times 6.022 \times 10^{23}}{2}$$

$= 92.73 \quad \approx 93 \text{ g mol}^{-1}$

80. (C)

$\lambda = 480 \text{ nm} = 480 \times 10^{-9}$ m, $\nu = ?$

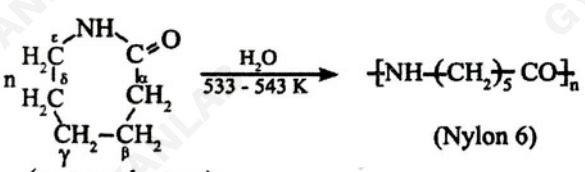
$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ ms}^{-1}}{480 \times 10^{-9} \text{ m}} = 6.25 \times 10^{14} \text{ Hz}$$

81. (B)

$$CH_3{-}CH_2{-}Cl \xrightarrow{AgNO_2} CH_3{-}CH_2{-}NO_2 + AgCl\downarrow$$

Chloroethane   Nitroethane

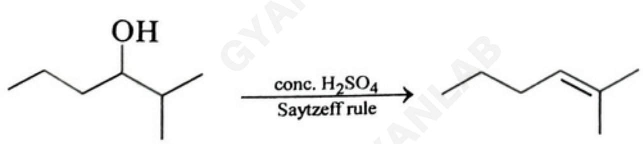
82. (D)



(ε - caprolactam)   (Nylon 6)

83. (D)

84. (A)



2–Methylhexan-3-ol   2-Methylhex-2-ene

85. (C)

86. (A)

In homologous series, each member differs from next member by a constant difference of one methylene group ($-CH_2-$).

$-CH_2- = 14 \text{ g mol}^{-1}$

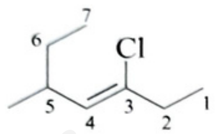
87. (D)

.... Ala–Gly–Ser–Tyr–Gly ....

← N-terminal   C-terminal →

88. (C)

89. (B)



3-Chloro-5-methylhept-3-ene

90. (A)

$$\overset{+1}{H_2}\overset{+2}{S} + \overset{+4}{N}\overset{-2}{O_2} \longrightarrow \overset{+1}{H_2}\overset{-2}{O} + \overset{+2}{N}\overset{-2}{O} + \overset{0}{S}$$

Reductant → H_2S

Oxidant → NO_2

91. (A)

92. (B)

Alkanes have lower boiling point as compared to amines of comparable molar mass.

93. (C)

As branching increases, boiling point decreases for isomers. Hence, tert-butyl alcohol has lowest boiling point due to small surface area.

94. (D)

NH_4NO_3 is a salt of strong acid, HNO_3 and weak base, NH_4OH.

95. (C)

96. (A)

97. (A)

$W_2 = 6$ g, $W_1 = 100$ g, $\Delta T_f = 0.93$ K, $K_f = 1.86$ K kg mol^{-1}, $M_2 = ?$

$$M_2 = \frac{1000\ K_f\ W_2}{\Delta T_f\ W_1} = \frac{1000\,g\,kg^{-1} \times 1.86\,K\ kg\,mol^{-1} \times 6\,g}{0.93\,K \times 100\,g}$$

$\therefore\ M_2 = 120$ g mol^{-1}

98. (C)

Kohlrausch theory is useful to calculate the molar conductivity of weak electrolytes.

99. (C)

A fcc unit cell has lattice points at the eight corners of the cell and at the centre of its six faces.

$\therefore\ 8 + 6 = 14$ lattice points

100. (D)

Section II

MATHEMATICS

101.(A)
Number of diagonals of 'n' sided polygons $= {}^nC_2 - n$

$\therefore \quad {}^nC_2 - n = 44$

$\frac{n!}{2!(n-2)!} - n = 44 \Rightarrow n(n-1) - 2n = 88$

$\therefore \quad n^2 - 3n - 88 = 0 \Rightarrow (n-11)(n+8) = 0 \Rightarrow n = 11 \qquad \ldots[n \in N]$

102.(B)
We hve $x - 1 = 2i \Rightarrow x^2 - 2x + 5 = 0 \qquad \ldots(1)$

$x^3 + 7x^2 - x + 16 = x(x^2 + 7x - 1) + 16$
$= x[(x^2 - 2x + 5) + (9x - 6)] + 16$
$= x[(0) + 9x - 6] + 16$
$= 9x^2 - 6x + 16$
$= 9(x^2 - 2x + 5) + 12x - 29$
$= 9(0) + 12(1 + 2i) - 29$
$= -17 + 24i$

103.(D)
$y^2 = ax^2 + bx + c$

Differentiatig w.r.t. x, we get

$2y\frac{dy}{dx} = 2ax + b \Rightarrow 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 2a$

$\therefore \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = a$

$\therefore \quad y^3\frac{d^2y}{dx^2} = -(ax^2 + bx + c)\left[\left(\frac{2ax+b}{2}\right)^2 - a\right] \qquad \ldots(1)$

R.H.S. of eq. (1) is a function of 'x' only.

104.(C)
The circle passes through the points (0, 0), (–2, 0) and (0, 3).

We have $x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore \quad c = 0 \qquad \ldots[\because$ It passes through (0, 0)]

$\therefore \quad x^2 + y^2 + 2gx + 2fy = 0$

$\therefore \quad (-2)^2 + 2g(-2) = 0 \Rightarrow 4 - 4g = 0 \Rightarrow g = 1$

Also $(3)^2 + 2f(3) = 0 \Rightarrow 6f = -9 \Rightarrow f = \frac{-3}{2}$

Thus centre $\equiv \left(-1, \frac{3}{2}\right)$ and radius $= \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

Hence required equation of circle is

$x^2 + y^2 + 2(1)x + 2\left(\frac{-3}{2}\right)y + 0 = 0$ i.e. $x^2 + y^2 + 2x - 3y = 0$

105.(B)

We have $f(x) = -x + 3, \text{ if } x \le -3$

$= -2x \text{ , if } -3 < x < 3$

$= 6x + 2, \text{ if } x \ge 3$

$\lim_{x \to -3^-} f(x) = -(-3) + 3 = 6$ and $\lim_{x \to -3^+} f(x) = -2(-3) = 6$ and $f(-3) = -(-3) + 3 = 6$

Thus f(x) is continuous at $x = -3$

$\lim_{x \to 3^-} f(x) = -2(3) = -6$ and $\lim_{x \to 3^+} f(x) = 6(3) + 2 = 20$

Thus $\lim_{x \to 3^-} f(x) \ne \lim_{x \to 3^+} f(x)$

Thus f(x) is not continuous at $x = 3$.

106.(D)

$y(1 + \log x) = \left(\log x^x\right)\frac{dy}{dx}$

$\therefore \int \frac{(1+\log x)}{\left(\log x^x\right)} = \int \frac{dy}{y}$

We know that $\frac{d}{dx}\left(\log x^x\right) = (1 + \log x)\,dx$

$\therefore \log|\log x^x| = \log|y| + \log c$

We have $y(e) = e^2$

$\therefore \log|\log e^e| = \log|e^2| + \log c$

$\therefore \log|e \log e| = 2 \log|e| + \log c$

$\therefore 1 = 2 + \log c \Rightarrow \log c = -1$

$\therefore \log|\log x^x| = \log|y| - \log e$

$\therefore \log|x \log x| + \log e = \log|y|$

$\therefore e\,x \log x - y = 0$

107.(B)

$\sim(p \to q) \leftrightarrow (r \wedge s)$

$\equiv \sim(T \to T) \leftrightarrow (F \wedge F)$

$= \sim(T) \leftrightarrow F \equiv F \leftrightarrow F \equiv T$

Also $(\sim p \to q) \wedge (r \leftrightarrow s)$

$\equiv (\sim T \to T) \wedge (F \leftrightarrow F)$

$\equiv (F \to T) \wedge (T) \equiv T \wedge T \equiv T$

108.(D)

Half life period of bismath is 5 days.

Initial mass = 1000 mg.

$\therefore$ Mass left after 5 days = 500 mg

Mass left after 10 days = 250 mg
Mass left after 15 days = 125 mg
Mass left after 20 days = 62.5 mg
Mass left after 25 days = 31.25 mg
Mass left after 30 days = 15.625 mg

109.(B)

$2ab\sin\frac{1}{2}(A+B-C)$

$= 2ab\sin\frac{1}{2}[(\pi-C)-C] = 2ab\sin\left(\frac{\pi-2C}{2}\right) = 2\,ab\sin\left(\frac{\pi}{2}-C\right) = 2ab\cos C$

$= 2ab\left(\frac{a^2+b^2-c^2}{2ab}\right) = a^2+b^2-c^2$

110.(B)

$\tan 3A = \text{ta}(2A+A)$

$= \frac{\tan 2A + \tan A}{1-\tan 2A \tan A}$

$\therefore \tan 3A - \tan 3A \tan 2A \tan A = \tan 2A + \tan A$

$\therefore \tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$

111.(C)

$ax^2 + 2hxy + by^2 = 0$

We have $m_1 + m_2 = \frac{-2h}{b}$ and $m_1 m_2 = \frac{a}{b}$

Also $m_1 = 2m_2$ ...(given)

$\therefore 3m_2 = -\frac{2h}{b} \Rightarrow m_2 = \frac{-2h}{3b}$ and $2m_2^2 = \frac{a}{b}$

$\therefore 2\left(\frac{-2h}{3b}\right)^2 = \frac{a}{b} \Rightarrow \frac{8h^2}{9b^2} = \frac{a}{b} \Rightarrow \frac{h^2}{ab} = \frac{9}{8}$

112.(D)

$\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

$\therefore \frac{dy}{dx} = \sin(x+y)$

Put $x + y = t \Rightarrow 1+\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} - 1 = \sin t \Rightarrow \frac{dt}{dx} = 1 + \sin t$

$\therefore \int\frac{dt}{1+\sin t} = \int dx$

$$\therefore \int \frac{(1-\sin t)dt}{1-\sin^2 t} = \int dx \Rightarrow \int \frac{1-\sin t}{\cos^2 t} dt = \int dx \Rightarrow \int (\sec^2 t - \sec t \tan t) dt = \int dx$$

$$\therefore \tan t - \sec t = x + c$$

$$\therefore \tan(x+y) - \sec(x+y) = x + c$$

113.(B)

We have $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$$\therefore |A| = (2-6) + (0-2) = -6$$

$$\text{adj } A = \begin{bmatrix} -4 & 3 & -2 \\ 2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}^T$$

$$\therefore A^{-1} = \frac{\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}}{-6}$$

$$\therefore |A^{-1}| = \frac{1}{(-6)^3}[(-4)(-6) - 2(0) - 2(-6)] = \frac{36}{-216} = \frac{-1}{6}$$

This problem can be alternately solved as follows :

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} = (-4) + (-2) = -6$$

We know that $|A^{-1}| = \frac{1}{|A|}$

$$\therefore |A^{-1}| = \frac{1}{-6}$$

114.(D)

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy} = \frac{x}{y} + \frac{y}{x}$$

Let $\frac{x}{y} = v \Rightarrow y = \frac{x}{v}$

$$\therefore \frac{dy}{dx} = \frac{v - x\left(\frac{dv}{dx}\right)}{v^2}$$

$$\therefore \frac{v - x\left(\frac{dv}{dx}\right)}{v^2} = v + \frac{1}{v} \Rightarrow \frac{1}{v} - \left(\frac{x}{v^2}\right)\frac{dv}{dx} = v + \frac{1}{v}$$

$$\therefore \int \frac{dv}{v^3} = -\int \frac{dx}{x}$$

$$\therefore \frac{-1}{2v^2} = -\log x - c \Rightarrow \frac{1}{2v^2} = \log x + c$$

$$\therefore \frac{y^2}{2x^2} = \log x + c \Rightarrow y^2 = x^2 \log x^2 + 2x^2 c$$

We have $y(1) = -2$

$\therefore 4 = 0 + 2c \Rightarrow c = 2$

$\therefore y^2 = x^2 \log x^2 + 4x^2$

115.(D)

$3x + 1 = 6y - 2 = 1 - z$

$$\therefore 3\left(x+\frac{1}{3}\right) = 6\left(y-\frac{2}{6}\right) = -(z-1)$$

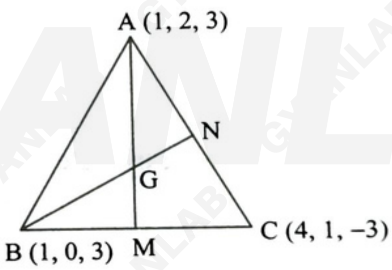
$$\therefore \frac{\left(x+\frac{1}{3}\right)}{\left(\frac{1}{3}\right)} = \frac{\left(y-\frac{1}{3}\right)}{\left(\frac{1}{6}\right)} = \frac{(z-1)}{(-1)} \Rightarrow \text{d.r.s. are } 2, 1, -6$$

Thus $\bar{r} = \left(-\frac{1}{3}\hat{i}+\frac{1}{3}\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}-6\hat{k}\right)$

116.(B)

Refer figure.

Let M be the mid point of BC and N be the midpoint of AC.



$$\therefore M = \left(\frac{5}{2}, \frac{1}{2}, 0\right)$$

We know that centroid G divides AM internally in the ratio 2 : 1

$$\therefore G = \frac{(1)(1)+(2)\left(\frac{5}{2}\right)}{2+1}, \frac{(1)(2)+(2)\left(\frac{1}{2}\right)}{2+1}, \frac{(1)(3)+0}{2+1}$$

$G = (2, 1, 1)$

117.(A)

Let $f(x) = \dfrac{x \sin x}{1+\cos^2 x}$

$$f(-x) = \frac{(-x)\sin(-x)}{1+\cos^2 x} = \frac{x \sin x}{1+\cos^2 x}$$

$\therefore f(x) = f(-x) \Rightarrow f(x)$ is an even function

Let $I = \displaystyle\int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$$\therefore \; I = 2\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx \qquad \ldots(1)$$

$$= 2\int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+[\cos(\pi - x)]^2}dx = 2\int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx \qquad \ldots(2)$$

Eq (1) + (2) gives,

$$2I = 2\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Put $\cos x = t \Rightarrow \sin x\, dx = -dt$. Also when $x = 0$, $t = 1$ and when $x = \pi$, $t = -1$

$$\therefore \; 2I = 2\pi\int_1^{-1} \frac{-dt}{1+t^2}$$

$$= 2\pi\int_{-1}^{1} \frac{dt}{1+t^2} = 4\pi\int_0^{1} \frac{dt}{1+t^2} = 4\pi\left[\tan^{-1} t\right]_0^1 = 4\pi\left(\frac{\pi}{4}\right) = \pi^2$$

$$\therefore \; I = \frac{\pi^2}{2}$$

118.(D)

Let $I = \int e^x\left(\frac{x-1}{x^2}\right)dx = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = e^x\left(\frac{1}{x}\right) + c$

119.(C)

$y^2 - 9xy + 18x^2 = 0$

$\therefore \; (y - 3x)(y - 6x) = 0$

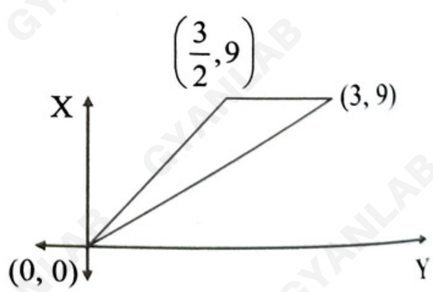
Thus three lines forming triangle are

$y = 3x$, $y = 6x$, $y = 9$

Their point of intersections are $(0, 0)$, $(3, 9)$, $\left(\frac{3}{2}, 9\right)$

$\therefore$ Area of triangle

$= \frac{1}{2} \times \frac{3}{2} \times 9 = \frac{27}{4}$ sq. units



120.(D)

$y^2 = 2(x - 3)$

$\therefore \; 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}$

$\therefore$ Slope of normal $= -y$ and as per condition given,

$-y = 2 \Rightarrow y = -2$

$\therefore \; (-2)^2 = 2(x - 3) \Rightarrow x = 5 \Rightarrow$ point is $(5, -2)$

121.(C)

We have $P(A \cup B) = \frac{5}{6}$, $P(A) = \frac{1}{6}$, $P(B) = \frac{2}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{5}{6} = \frac{1}{6} + \frac{2}{3} - P(A \cap B) \Rightarrow P(A \cap B) = 0$

Thus A and B are mutually exclusive events.

122.(C)

Refer figure.

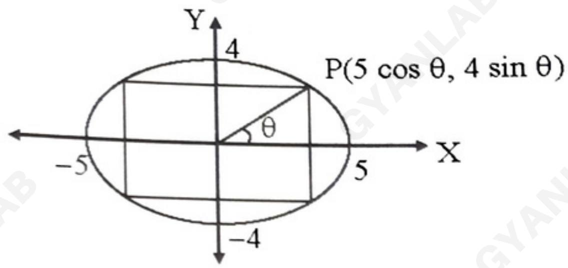
Length of rectangle $= 10 \cos \theta$ and

breath of reactangle $= 8 \sin \theta$.

$\therefore$ Area of rectangle $= (10 \cos \theta)(8 \sin \theta)$
$= 40 (\sin 2\theta)$

Maximum area will occur when $\sin 2\theta = 1$

$\therefore \sin 2\theta = \sin \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$



$\therefore P = \left(\frac{5}{\sqrt{2}}, \frac{4}{\sqrt{2}}\right) \Rightarrow$ Dimensions of rectangle are $5\sqrt{2}$, $4\sqrt{2}$.

123.(A)

Refer figure.

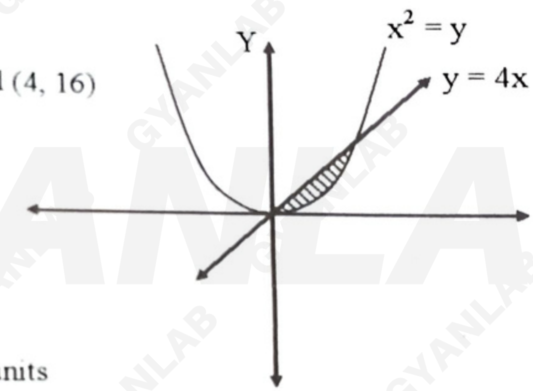
Required area is shaded.

Point of intersection of given curves are (0, 0) and (4, 16)

$\therefore A = \int_0^4 (4x - x^2)\,dx$

$= 4\int_0^4 x\,dx - \int_0^4 x^2\,dx = 4\left[\frac{x^2}{2}\right]_0^4 - \left[\frac{x^3}{3}\right]_0^4$

$= 2(16) - \frac{(4)^3}{3} = 32 - \frac{64}{3} = \frac{32}{3}$ sq. units



124.(C)

$\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2}$

$= \lim_{x \to 0} \frac{\left[-2\sin\frac{(m+n)x}{2}\sin\frac{(m-n)x}{2}\right]}{x^2}$

$= -2\lim_{x \to 0}\left[\frac{\sin\left(\frac{m+n}{2}\right)x}{\left(\frac{m+n}{2}\right)x} \times \left(\frac{m+n}{2}\right)\right]\left[\sin\frac{\left(\frac{m-n}{2}\right)x}{\left(\frac{m-n}{2}\right)x} \times \left(\frac{m-n}{2}\right)\right]$

$= (-2)\left(\frac{m+n}{2}\right)\left(\frac{m-n}{2}\right) = \frac{m^2 - n^2}{-2} = \frac{n^2 - m^2}{2}$

125.(B)

Let $d_1 = 3\hat{i} - \hat{j} - 2\hat{k}$ and $d_2 = -\hat{i} + 3\hat{j} - 3\hat{k}$

$\therefore |d_1| = \sqrt{14}$ and $|d_2| = \sqrt{19}$

Also $d_1 \times d_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ -1 & 3 & -3 \end{vmatrix} = 9\hat{i} + 11\hat{j} + 8\hat{k}$

$\therefore |d_1 \times d_2| = \sqrt{81+121+64} = \sqrt{266}$

$|d_1 \times d_2|^2 = |d_1|^2 \, |d_2|^2 \sin^2\theta$

$\therefore 266 = (14)(19)\sin^2\theta$

$\therefore \sin^2\theta = 1 \Rightarrow \sin\theta = 1 \quad \ldots[\because 0 < \theta \le \frac{\pi}{2}]$

Area of parallelogram $= \frac{1}{2}|d_1||d_2|\sin\theta$

$= \frac{\sqrt{266}}{2}$

126.(A)

$\bar{a} + \bar{b} = -\bar{c} \Rightarrow |\bar{a} + \bar{b}|^2 = |\bar{c}|^2$

$\therefore |\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}|\cos\theta = |\bar{c}|^2$

$\therefore (3)^2 + (5)^2 + 2(3)(5)\cos\theta = (7)^2$

$\therefore \cos\theta = \frac{49-34}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

127.(D)

The vertices of triangle ABC are

A = (2, 0, 0); B = (0, 3, 0); C = (0, 0, 4)

$\overline{AB} = -2\hat{i} + 3\hat{j}$ and $\overline{AC} = -2\hat{i} + 4\hat{k}$

$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 0 \\ -2 & 0 & 4 \end{vmatrix} = \hat{i}(12) - \hat{j}(-8) + \hat{k}(6) = 12\hat{i} + 8\hat{j} + 6\hat{k}$

$\therefore A(\Delta ABC) = \frac{1}{2}|\overline{AB} \times \overline{AC}|$

$= \frac{1}{2}\left(\sqrt{144+64+36}\right)$

$= \sqrt{61}$ sq. units

128.(B)

We have $k + 3k + 5k + 7k + 8k + k = 1 \Rightarrow k = \frac{1}{25}$

$\therefore P(2 \le x < 5) = \frac{1}{25}(3+5+7) = \frac{15}{25} = \frac{3}{5}$

129.(D)

$$V = \frac{4}{3}\pi r^3$$

$$\therefore \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\therefore 8 = 4\pi(2)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{2\pi} \text{ cm/sec}$$

130.(D)

We have $|\bar{u}| = 2$ and $\cos\alpha = 60° = \frac{1}{2}$ and $\cos\beta = \cos 120° = -\frac{1}{2}$

Now $\cos^2\gamma = 1 - \left(\frac{1}{4}+\frac{1}{4}\right) = \frac{1}{2} \Rightarrow \cos\gamma = \pm\frac{1}{\sqrt{2}}$

Thus direction ratio of $\bar{u}$ are $1, -1, \pm\sqrt{2}$

$$\therefore \bar{u} = 2\left(\hat{i} - \hat{j} \pm \sqrt{2}\,\hat{k}\right)$$

131.(B)

$$A = \begin{bmatrix} k & 2 \\ -2 & -k \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} k & 2 \\ -2 & -k \end{vmatrix} = -k^2 + 4$$

When $-k^2 + 4 = 0$, we get $k = \pm 2$

132.(B)

Angles A, B, C of ΔABC are in A.P.

$\therefore \angle B = 60° \Rightarrow \angle A + \angle C = 120°$

Also $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$ ...(let)

$\therefore a = k\sin A$ & $c = k\sin C$

$$\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$$

$$= \frac{k\sin A}{k\sin C}(2\sin C\cos C) + \frac{k\sin C}{k\sin A}(2\sin A\cos A)$$

$$= 2\sin A\cos C + 2\cos A\sin C = 2(\sin A\cos C + \cos A\sin C)$$

$$= 2\sin(A + C) = 2\sin(120°) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

133.(D)

A coin is tossed 3 times.

$\Rightarrow n(S) = 8$

Possibilities are : (1H, 2T), (2H, 1T), (3H, 0T), (0H, 3T)

Thus values of X can be 1 and 3.

Now (1H, 2T) can occur in 3 ways. Also (2H, 1T) can occur in 3 ways.

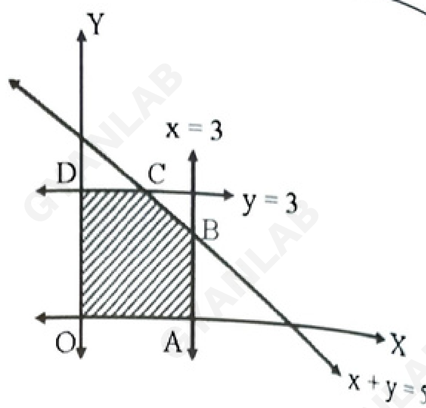
$$\therefore P(X = 1) = \frac{6}{8} = \frac{3}{4}$$

134.(B)

Refer figure

Required area is shaded.

Vertices of the required region are 0(0, 0); A(3, 0); B(3, 2); C(2, 3); D(0, 3)



We have to maximize objective function

$z = 10x + 25y$

$\therefore\ z_{(O)} = 0 + 0 \quad = 0$

$z_{(A)} = 30 + 0 \quad = 30$

$z_{(B)} = 30 + 50 \quad = 80$

$z_{(C)} = 20 + 75 \quad = 95$

$z_{(D)} = 0 + 75 \quad = 75$

135.(A)

We will make truth table

p	q	~p	~q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$	$1 \wedge 4$	$2 \vee 9$	$2 \wedge 3$	$10 \vee 11$
1	2	3	4	5	6	7	8	9	10	11	12
T	T	F	F	T	T	T	T	F	T	F	T
T	F	F	T	T	F	F	F	T	T	F	T
F	T	T	F	T	F	T	F	F	T	T	T
F	F	T	T	F	F	T	T	F	F	F	F

Entries in columns 5 and 12 are identical.

136.(C)

Let A = (x, 0, 0); B = (0, y, 0); C = (0, 0, z).

Centroid of ΔABC is (1, 2, 3)

$\therefore\ \frac{x}{3} = 1,\ \frac{y}{3} = 2,\ \frac{z}{3} = 3 \Rightarrow (x, y, z) = (3, 6, 9).$

Hence equation of required plane is

$\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$

137.(C)

Let the numbers be x, y, z.

$x + y + z = 6$ ...(1)

$x + 3z = 7$ ...(2)

$3x + y + z = 12$ ...(3)

Eq. (3) – (1), gives

$2x = 6 \Rightarrow x = 3$

From eq. (2), we get

$3 + 3z = 7 \Rightarrow z = \frac{4}{3}$

From eq. (1), we get

$3+y+\frac{4}{3} = 6 \quad \Rightarrow \quad y = \frac{5}{3}$

$\therefore \ xyz = (3)\left(\frac{5}{3}\right)\left(\frac{4}{3}\right) = \frac{20}{3}$

Note : This problem can be alternatively solved as follows.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 6 \end{bmatrix}$$

$\therefore \ x+y+z = 6 \quad \ldots(1)$

$x+3z = 7 \quad \ldots(2)$

$2x = 6 \quad \ldots(3)$

$\therefore \ x = 3 \quad \ldots[\text{From } (3)]$

$\therefore \ 3+3z = 7 \quad \Rightarrow \quad z = \frac{4}{3} \quad \ldots[\text{From } (2)]$

$\therefore \ 3+y+\frac{4}{3} = 6 \quad \Rightarrow \quad y = \frac{5}{3} \quad \ldots[\text{From } (1)]$

138.(A)

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

$$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right] \quad = \tan^{-1}\left[-\tan\frac{\pi}{6}\right]+\cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]+\cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] \quad = -\frac{\pi}{6}+\frac{\pi}{6} = 0$$

139.(D)

$$\text{Let} \quad I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$$

$$= \int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1+x(1-x)}\right]dx \quad = \int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1-(x-1)(x)}\right]dx$$

$$= \int_0^1 \left[\tan^{-1}x+\tan^{-1}(x-1)\right]dx \quad = \int_0^1 \tan^{-1}x\,dx+\int_0^1 \tan^{-1}(x-1)\,dx$$

$$= \left[\left[x\tan^{-1}x\right]_0^1-\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx\right]+\left[\left[x\tan^{-1}(x-1)\right]_0^1-\frac{1}{2}\int_0^1\frac{2x-2+2}{1+(x-1)^2}dx\right]$$

$$= \left(\frac{\pi}{4}\right) - \frac{1}{2}\left[\log\left|1+x^2\right|\right]_0^1 + 0 - \frac{1}{2}\int_0^1 \frac{2x-2}{1+(x-1)^2}dx - \int_0^1 \frac{dx}{1+(x-1)^2}$$

$$= \left(\frac{\pi}{4}\right) - \frac{1}{2}\log 2 - \frac{1}{2}\left[\log\left|1+(x-1)^2\right|\right]_0^1 - \left[\tan^{-1}(x-1)\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2 - \frac{1}{2}(0 - \log 2) - \left[0 - \tan^{-1}(-1)\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2 + \frac{1}{2}\log 2 - \frac{\pi}{4} = 0$$

140.(D)

We have $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2at}{1+t^2}$

Put $t = \tan\theta \Rightarrow x = \cos 2\theta$ and $y = a\sin 2\theta$

$\frac{dx}{d\theta} = -2\sin 2\theta$ and $\frac{dy}{d\theta} = 2a\cos 2\theta$

$$\therefore \frac{dy}{dx} = \frac{2a\cos 2\theta}{-2\sin 2\theta} = \frac{-a}{\tan 2\theta} = \frac{-a}{\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$$

$$= \frac{-a}{\left(\frac{2t}{1-t^2}\right)} = \frac{-a(1-t^2)}{2t} = \frac{a(t^2-1)}{2t}$$

141.(D)

Let $I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

When $x = \tan\theta$, $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$ and $dx = \sec^2\theta\, d\theta$

$$\therefore I = \int 2\theta\sec^2\theta\, d\theta$$

$$= 2\int \theta\sec^2\theta\, d\theta = 2\left[[\theta\tan\theta] - \int\tan\theta d\theta\right] = 2\left[\theta\tan\theta + \log|\cos\theta|\right] + c$$

$$= 2(\tan^{-1}x)(x) + 2\log\left|\sqrt{\frac{1}{1+\tan^2\theta}}\right| + c = 2x\tan^{-1}x + 2\log\left|\sqrt{\frac{1}{1+x^2}}\right| + c$$

$$= 2x\tan^{-1}x + 2\log\left|1+x^2\right|^{-\frac{1}{2}} + c = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$$

142.(B)

$f(x) = \sqrt{x-1} + \sqrt{6-x}$

Here f(x) is defined when

$x - 1 \geq 0$ and $6 - x \geq 0$ i.e.

$x \geq 1$ and $x - 6 \leq 0$ i.e. $x \geq 1$ and $x \leq 6$

$\therefore$ Domain of f(x) is [1, 6]

143.(D)

$$y = mx + \frac{4}{m} \quad \ldots(1)$$

$$\therefore \frac{dy}{dx} = m$$

Substituting value of m in equation (1), we get

$$y = \left(\frac{dy}{dx}\right)x + \frac{4}{\left(\frac{dy}{dx}\right)}$$

$$\therefore y\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right)^2 x + 4$$

144.(A)

We find that each element of new data is 2 times of each element in old data.

Variance of old data $= 23.33$

$\therefore$ Variance of new data $= (23.33)(2)^2 = 93.32$

145.(D)

$$y = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$$

$$= \tan^{-1}\left[\sqrt{\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right] = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

146.(A)

We have $np = 18$ and $npq = 12$

$$\therefore q = \frac{12}{18} = \frac{2}{3} \quad \Rightarrow \quad p = 1-\frac{2}{3} = \frac{1}{3}$$

$$\therefore n\left(\frac{1}{3}\right) = 18 \quad \Rightarrow \quad n = 54$$

147.(A)

We have lines $\bar{r} = (2\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + \hat{j} - 5\hat{k})$

Let $\bar{a} = 2\hat{i} - \hat{j}$ and $\bar{b} = \hat{i} - \hat{j} + 2\hat{k}$

$\therefore \ \overline{AB} = -\hat{i} + 2\hat{k}$

Vector perpendicular to given lines is

$$\bar{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & 1 & -5 \end{vmatrix} = \hat{i}(-5+3) - \hat{j}(-10+6) + \hat{k}(2-2) = -2\hat{i} + 4\hat{j}$$

Shortest distance between given lines

$$= \overline{AB} \cdot \frac{\bar{n}}{|\bar{n}|} = \frac{(-\hat{i} + 2\hat{k}) \cdot (-2\hat{i} + 4\hat{j})}{\sqrt{(-2)^2 + (4)^2}} = \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$$

148.(D)

Slope of line AB $= \dfrac{-5-3}{6+2} = \dfrac{-8}{8} = -1$

Mid point of AB $= \dfrac{-2+6}{2}, \dfrac{3-5}{2} = (2, -1)$

Equation of perpendicular bisector of AB is $(y + 1) = 1(x - 2)$ i.e. $x - y = 3$

149.(A)

$$[\bar{a} + \bar{b} + \bar{c} \quad \bar{b} - \bar{a} \quad \bar{c}]$$
$$= (\bar{a} + \bar{b} + \bar{c}) \cdot [(\bar{b} - \bar{a}) \times \bar{c}]$$
$$= (\bar{a} + \bar{b} + \bar{c}) \cdot [(\bar{b} \times \bar{c}) - (\bar{a} \times \bar{c})]$$
$$= \bar{a} \cdot (\bar{b} \times \bar{c}) - \bar{b} \cdot (\bar{a} \times \bar{c}) = \bar{a} \cdot (\bar{b} \times \bar{c}) + \bar{b}(\bar{c} \times \bar{a})$$
$$= 2\bar{a} \cdot (\bar{b} \times \bar{c}) = 2(1)(2)(3) = 12$$

150.(C)

Let $I = \int \dfrac{\sec^8 x}{\operatorname{cosec} x} dx$

$$= \int \left(\frac{\sin x}{\cos x}\right)(\sec^6 x)(\sec x)\, dx$$

$$= \int (\sec^6 x)(\sec x \tan x)\, dx$$

Put $\sec x = t \Rightarrow \sec x \tan x\, dx = dt$

$$\therefore \ I = \int t^6\, dt = \frac{t^7}{7} + c = \frac{\sec^7 x}{7} + c$$